

DILLA UNIVERSITY
DEPARTMENT OF MATHEMATICS

Computer Algebra Exercise 1
due on Nov 21, 2017, 8:30 AM

1. (a) Implement the Euclidean Algorithm for computing the greatest common divisor in \mathbb{Z} . Test your implementation at examples.
(b) Use your implementation to cancel

$$\frac{90189116021}{18189250063}$$

2. Let p be a prime and $\mathbb{F}_p = \mathbb{Z}/p$ the field with p elements.
 - (a) Use an analogue of the sieve of Eratosthenes to find all irreducible polynomials in $\mathbb{F}_2[x]$ of degree ≤ 3 .
 - (b) Factor $x^5 + x^2 + x + 1 \in \mathbb{F}_2[x]$ into a product of irreducible polynomials.
 - (c) Determine all elements of $K = \mathbb{F}_2[x]/(x^2 + x + 1)$, the addition table of K , and the multiplication table of K . Prove that K is a field.
3. Write a procedure to compute

$$\pi(x) = |\{p \leq x \mid p \in \mathbb{N} \text{ prime}\}|$$

for $x > 0$.

4. Write a procedure to compute $n!$ for any $n \in \mathbb{Z}_{\geq 1}$.
5. Write a procedure which returns an n -th Fibonacci number.

Note: Write your procedure in Singular.