Dilla University

College of Natural and Computational Science

Department of Mathematics

Course Title: **Algebraic Geometry** Course Code: Math 713

Course hrs: 3 Credit hr: 3hrs Tutorial: 2hrs

Course Description: The course covers interrelation of Geometry, Algebra and algorithms, Groebner bases, Elimination theory, Polynomial and rational functions on a variety, the algebra–geometry dictionary.

Course Objectives: On completion of the course successful students will be able to:

- comprehend the concept of algebraic geometry
- understand the relationship between algebra and geometry
- solve problems in algebraic geometry
- perform parameterization of affine varieties
- understand Groebner bases and their properties
- apply Groebner bases
- find sums, products, and intersections of ideals
- decompose a given variety into irreducible varitties

Chapter 1: Geometry, Algebra and Algorithms

1.1 Polynomials and affine space

1.2 Affine varieties

1.3 Parametrizations of affine varieties

1.4 Ideals

1.5 Polynomials of one variable

Chapter 2: Groebner Bases

2.1 Introduction

2.2 Orderings on the monomials in $k[x1, . . . , xn]$

2.3 A division algorithm in $k[x1, . . . , xn]$

2.4 Monomial ideals and Dickson's Lemma

2.5 The Hilbert basis theorem and Groebner bases

2.6 Properties of Groebner bases

2.7 Buchberger's algorithm

2.8 First Applications of Groebner bases

2.9 Improvements on Buchberger's algorithm (Optional)

Chapter 3: Elimination Theory

3.1 The Elimination and Extension Theorems
3.2 The Geometry of Elimination
3.3 Implicitization
3.4 Singular Points and Envelopes
3.5 Unique factorization and resultants
3.6 Resultants and Extension Theorem

Chapter 4: The Algebra-Geomerty Dictionary

4.1 Hilbert's Nullstellensatz
4.2 Radical ideals and the ideal-variety correspondence
4.3 Sums, products and intersections of ideals
4.4 Zariski closure and quotients of ideals
4.5 Irreducible varieties and prime ideals
4.6 Decomposition of a variety into irreducibles
4.7 Primary decomposition of ideals (Optional)

Chapter 5: Polynomial and Rational Function on a Variety

5.1 Polynomial mappings
5.2 Quotients of Polynomial Rings
5.3 Algorithmic computations in $k[x1, \ldots, xn]/I$
5.4 The Coordinate ring of an affine variety
5.5 Rational Functions on a variety
5.6 Proof of the Closure theorem (optional)

Mode of Assessment:

- Assignment: 20%
- Mid exam: 30%
- Final exam: 50%

Text book: David Cox, John Little, Donal O'Shea , **Ideals Verities and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra**, 3rd Edition Springer, 2007.

Rereferences

1. M.Atiyah, I. G. Macdonald, Introduction to Commutative Algebra, Perseus Books 1999.
2. D. Eisenbud, Commutative Algebra with a View Toward Algebraic Geometry, Springer 2007.
3. G. -M. Greuel G. Pfister, A Singular Introduction to Commutative Algebra, Springer 2007.
4. E. Kunz, Introduction to Commutative Algebra and Algebraic Geometry, Birkhöuser 1985.
5. T.Y. Lam, Lectures on Modules and Rings, Springer 1998.