DILLA UNIVERSITY
DEPARTMENT OF MATHEMATICS

Advanced Linear Algebra Exercise 2
due on Dec 7, 2017, 8:30 AM

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1. Let $\tau : \mathbb{R}^3 \to \mathbb{R}^2$ be the projection defined for any $u = (x, y, z) \in \mathbb{R}^3$ by $\tau(u) = (x, y, 0)$. Show that τ a L.T.

2. Define $\tau : \mathbb{R}^3 \to \mathbb{R}^2$ by $\tau(x, y, z) = (z - x, x + y)$.
   a) Compute $\tau(e_1), \tau(e_2)$ and $\tau(e_3)$.
   b) Show τ is a L.T.
   c) Show $\tau(x, y, z) = x\tau(e_1) + y\tau(e_2) + z\tau(e_3)$.

3. Let P_n be the set of polynomials in x of degree at most n. Define the function $D : P_3 \to P_2$ by $D(f) = \mathrm{d}f/\mathrm{dx}$. Show that D is a L.T.

4. Let V and W be vector spaces over over a field F and let $\mathcal{B} = \{v_i \mid i \in I\}$ is a basis for V. Then for any $\tau \in \mathcal{L}(V, W)$, we have $\mathrm{im}(\tau) = \langle \tau(\mathcal{B}) \rangle$.

5. Let τ be a L.T from a vector space V into a vector space W. Then
   i) $\tau(0) = 0$.
   ii) $\tau(-v) = -\tau(v)$ for all $v \in V$.
   iii) $\tau(u - v) = \tau(u) - \tau(v)$ for all $u, v \in V$.
   iii)
   $$\tau\left(\sum_{k=1}^{n} a_k v_k\right) = \sum_{k=1}^{n} a_k \tau(v_k)$$
   for all $v_1, \ldots, v_k \in V$.