DILLA UNIVERSITY
DEPARTMENT OF MATHEMATICS

Advanced Linear Algebra Exercise 1
due on Nov 30, 2017, 8:30 AM

1. Let $\mathbb{R}$ be the set of real numbers, and let

$$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; a, b, c, d \in \mathbb{R} \right\} = \mathbb{R}^{2 \times 2}$$

   be the set of all 2×2 matrices with entries in $\mathbb{R}$.

   i) Show that V is an $\mathbb{R}$-vector space.

   ii) Show that the subset

$$U = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \;\middle|\; a - 2b + 3c = 0 \right\}$$

   of V is a subspace of V.

   iii) Find $\dim V$ and $\dim U$.

2. Let $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ be the field generated by elements of the form $a + b\sqrt{2} + c\sqrt{3}$, where $a, b, c \in \mathbb{Q}$, that is,

$$\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \{ a + b\sqrt{2} + c\sqrt{3} \mid a, b, c \in \mathbb{Q} \}.$$

   (i) Prove that $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is a vector space of dimension 4 over $\mathbb{Q}$.

   (ii) Find a basis for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

3. Which of the following sets are subspaces of $\mathbb{R}^3$? If the set is indeed a subspace, find a basis for the subspace and compute its dimension.

   a) $U = \{ (x, y, z) \mid 3x - 5y + 2z = 0 \}$

   b) $V = \{ (x, y, z) \mid 2x - z^2 = 0 \}$