# Dilla University

## College of Natural and Computational Science

## Department of Mathematics

Course Title: Advanced Linear Algebra Course Code: Math 609

Credit hr: 3hrs Tutorial: 2hrs Course hrs: 3

**Aims of the course:** The aim of this course is to introduce advanced linear algebra which is application oriented and hence will be useful to students who want to take a course on Numerical Analysis, Operations research or other applied areas that require computational linear algebra.

**Course Description:** The course covers linear transformations and their representations via vector space, matrices, kernel and range, duality and the interpretation of the adjoint matrix, and matrix theory (eigenvalues and eigenvectors, diagonalization, traingularization, canonical forms).

**Course Objectives:** On completion of the course successful students will be able to:

- comprehend basic properties of vectors spaces
- comprehend the notion of linear independence
- comprehend the notion of bases and dimensions
- comprehend fundamental properties of linear transformations
- grasp the concept of eigenspaces
- find eigenvalues and eigenvectors of a square matrix numerically
- diagonalize a matrix when this is possible
- solve problems involving understanding of linear algebra concepts
- apply important techniques such as Jordan canonical forms in other mathematical disciplines

**Chapter 1: VECTOR SPACES**

1.1 The axioms of a vector space

1.2 Examples of different models of a vector space

1.3 Subspaces, linear combinations and generators

1.4 Linear dependence and independence of vectors

1.5 Bases and dimension of a vector space

1.6 Direct sum and direct product of subspaces

**Chapter 2: LINEAR TRANSFORMATIONS**

2.1 Definition of linear transformation and examples

2.2 The rank and nullity of a linear transformation

2.3 Algebra of linear transformation

2.4 Matrix representation of a linear transformation

**Chapter 3: DIAGONALIZATION AND INNER PRODUCT SPACES**

3.1 Eigenvalues and Eigenvectors, Diagonalization

3.2 Block Matrices and their properties

3.3 Jordan Canonical Forms

3.4 Inner products

3.5 Length, angle and orthogonality in inner product spaces

3.6 Orthonormal Bases

3.7 Gram-Schmidt orthogonalization process

3.8 Orthogonal and unitary Matrices

3.9 Properties of Symmetric and Hermitian Operators

3.10 Shur's Theorem

3.11 Diagonalization of a symmetric matrix and the Spectral Theorem

**Chapter 4: QUADRATIC FORMS**

4.1 Definition and examples of quadratic forms

4.2 Definiteness of Linear Operators

4.3 Testing Definiteness of Matrices and Linear Operators

4.4 Positive definite matrices and quadratic forms

4.5 Diagonalizing quadratic forms

4.6 Sylvester's laws of inertia

**Chapter 5: OPERATOR FACTORIZATION**

5.1 QR decomposition
5.2 Singular-Value decomposition
5.3 The Moore-Penrose inverse
5.4 Least square approximations

**Mode of Assessment:**

- Assignment: 20%
- Mid exam: 30%
- Final exam: 50%

**Text book:** Steven Roman; Advanced Linear Algebra, 2005**.**

**Rereferences**


1. Steven Roman; Advanced Linear Algebra, 2005.
2. S. Lang; Linear Algebra.
3. Gilbert Strang; Linear Algebra and Its Application,Thomson Learning, 1988.
4. J. S. Golan; The Linear Algebra a Beginning Grad. Student Ought to Know, 2007.
5. Kenneth Kuttler; An Introduction to Linear Algebra, 2004.
6. R. Baker, K. Kutter; Linear Algebra with Applications, World Scientific Publishing Co. 2014.