DILLA UNIVERSITY
DEPARTMENT OF MATHEMATICS

Algebra I Exercise 2
due on Nov 22, 2017, 8:30 AM

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1. Let Q_8 be the group (under ordinary matrix multiplication) generated by the complex matrices $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, where $i^2 = -1$. Show that Q_8 is a nonabelian group of order 8. The group Q_8 is called the *quaternion group*. [Hint: Observe that $BA = A^3B$, whence every element of Q_8 is of the form A^iB^j. Note also that $A^4 = B^4 = I$, where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity element of Q_8.]

2. Let S be a nonempty subset of a group G and define a relation on G by $a \sim b$ if and only if $ab^{-1} \in S$. Show that $\sim$ is an equivalence relation if and only if S is a subgroup of G.

3. Let $f : G \to H$ be a homomorphism of groups, A a subgroup of G, and B a subgroup of H. Show that:
   (a) $f^{-1}(B)$ is a subgroup of G.
   (b) $f(A)$ is a subgroup of H.

4. List all subgroups of $\mathbb{Z}_2 \oplus \mathbb{Z}_2$. Is $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ is isomorphic to $\mathbb{Z}_4$?