

DILLA UNIVERSITY
DEPARTMENT OF MATHEMATICS

Algebra I Exercise 1
due on Nov 22, 2017, 8:30 AM

1. Let a, b be elements of group G . Show that
 - i) $|a| = |a^{-1}|$,
 - ii) $|ab| = |ba|$, and
 - iii) $|cac^{-1}| = |a|$ for all $c \in G$
2. Prove that the following conditions on a group G are equivalent (**Abelian Relations**):
 - (i) G is abelian;
 - (ii) $(ab)^2 = a^2b^2$ for all $a, b \in G$;
 - (iii) $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$;
 - (iv) $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$ and all $a, b \in G$;
 - (v) $(ab)^n = a^n b^n$ for all three consecutive integers n and all $a, b \in G$; (Hint: show that (i) \Leftrightarrow (ii), (iii) and (i) \Rightarrow (iv) \Rightarrow (v) \Rightarrow (i).)
3. If $a^2 = e$ for all elements a of a group G , then prove that G is abelian. (**Groups of Involutions**)