DILLA UNIVERSITY
DEPARTMENT OF MATHEMATICS

Advanced Linear Algebra Exercise 1
due on Dec 14, 2018, 8:30 AM

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1. Let $\mathbb{R}$ be the set of real numbers, and let
$$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; a, b, c, d \in \mathbb{R} \right\} = \mathbb{R}^{2\times 2}$$
be the set of all 2×2 matrices with entries in $\mathbb{R}$. Show that V is an $\mathbb{R}$-vector space and show also that the subset
$$U = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \;\middle|\; a + 3b - 4c = 0 \right\}$$
of V is a subspace of V. Furthermore, find $\dim V$ and $\dim U$.

2. Which of the following sets are subspaces of $\mathbb{R}^3$? If the set is indeed a subspace, find a basis for the subspace and compute its dimension.
   a) $U = \{(x, y, z) \mid 7x - 2y + 10z = 0\}$
   b) $V = \{(x, y, z) \mid 2x - y^2 = 0\}$

3. Let $\tau : \mathbb{R}^3 \to \mathbb{R}^2$ be the projection defined for any $u = (x, y, z) \in \mathbb{R}^3$ by $\tau(u) = (x, y, 0)$. Show that τ a L.T.

4. Define $\tau : \mathbb{R}^3 \to \mathbb{R}^2$ by $\tau(x, y, z) = (z - x, x + y)$. Is τ a L.T? If so, compute $\tau(e_1), \tau(e_2)$ and $\tau(e_3)$ and show that $\tau(x, y, z) = x\tau(e_1) + y\tau(e_2) + z\tau(e_3)$.

5. Let V and W be vector spaces over over a field F and let $\mathcal{B} = \{v_i \mid i \in I\}$ is a basis for V. Then for any $\tau \in \mathcal{L}(V, W)$, we have $\operatorname{im}(\tau) = \langle \tau(\mathcal{B}) \rangle$.

6. Let τ be a L.T from a vector space V into a vector space W. Then
   i) $\tau(0) = 0$.
   ii) $\tau(-v) = -\tau(v)$ for all $v \in V$.
   iii) $\tau(u - v) = \tau(u) - \tau(v)$ for all $u, v \in V$.